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Vikash Raj
Assistant Professor,
Department of Physics, Lord Krishna College, Samastipur, Bihar, India

Corresponding Author:
Vikash Raj
Assistant Professor,
Department of Physics, Lord Krishna College, Samastipur, Bihar, India

TE scattering by perfectly conducting arbitrarily connected strips

Vikash Raj

Abstract
The analysis of objects with edges by means of integral-differential operators, requires the appropriate selection of the functional space where the solution has to be searched for. Mexiner derived contions for the electromagnetic field which in turn imply a proper edge behaviour of the currents induced on the scatter. Therefore, such conditions characterize the functional space to which the solution has to belong. In this paper the scattering by an arbitrary collection of perfectly conducting connected strips is analysed for TE incidence. The studied configurations include both closed and open polygonal cross-section cylinders, as well as more complicated structures in which more than two strips are connected at a point. The proposed method is very efficient as few expansion functions are needed in a Galerkin scheme. This is achieved by means of expansion functions factorizing the correct edge singularity of the electromagnetic field and ensuring the continuity of the current at connecting points.

Keywords: Conducting strips, electromagnetic scattering, Galerkin method.

Introduction
Many authors addressed the problem expanding the unknown current by means of orthogonal functions factorizing the correct edge behavior and satisfying the continuity condition of the current [2-10]. In particular in [7, 9] and [10] the use of Galerkin method in the spectral domain is proposed for the analysis of closed scatterers, to solve the EFIE for TM incidence, and the MFIE for TE incidence. The procedure is quickly convergent even when high accuracy is required and proved itself stable even at frequencies very close to the theoretical internal resonance frequencies, but the analysis is limited to closed scatterers for TE incidence, since MFIE cannot be used in the analysis of infinitesimally thin scatterers [1]. In this communication such a limit has been overcome, by suitably reducing the integral-differential equation for the electric field associated to the problem to an EFIE. It is worth nothing that the present formulation has been devised in such a way to preserve all the interesting characteristics mentioned above.

Theoretical formulation
In this communication TE scattering by arbitrary 2D perfectly conducting cylinders is analyzed, including closed (Fig.-1(a), 1(b)] polygonal cross-section cylinders, but also more complex structures in which more than two sides are connected at a point (Fig.- 1(c)) .
For TE incidence, only the transverse component of the current is present, and from Maxwell equations the transverse component along xi of the electric field is

$$E_j(x,y) = \frac{1}{j\omega\varepsilon}\frac{\partial}{\partial y_j}H_z(x,y)$$

$$= \frac{1}{2\omega\varepsilon}\frac{\partial}{\partial y_j}\sum_{i=1}^{L}\frac{\partial}{\partial y_j}\int_{-\infty}^{+\infty}\hat{J}_i(u)\frac{e^{-j|y_i|\sqrt{k^2-u^2}}}{\sqrt{k^2-u^2}}e^{-jux_i}\,du \qquad (1)$$

where $\hat{j}_1(u)$ is the Fourier transform of the current induced on ith side. An integral differential equation for the electric field can be obtained by imposing the total tangential component of the electric field to be vanishing onto the surface of the cylinder.
Although we are concerned in solving the problem in the spectral domain, it is useful to examine the spatial distribution of the currents with particular reference to the edge behavior, to have a better insight into the physics of the problem.

As a matter of fact, in many papers [2-10] the advantages, in terms of fast convergence and error control of using the Galerkin method with expansion functions such that the correct edge behavior of the electromagnetic field can be reconstructed, have been pointed out. With reference to the ith strip of a perfectly conducting cylinder, such edge behavior expressed in terms of the currents edge behavior is [1]

$$\frac{\partial}{\partial x_i} j_i(x_i) \underset{x_i \to a_i}{\sim} w_i^{\pm}(x_i) = \left(1 \mp \frac{x_i}{a_i}\right)^{\frac{\psi_i^{\pm}-\pi}{2\pi-\psi_i^{\pm}}} \qquad (2)$$

where $\psi_i^{\pm}$ is the angle at abscissa $x_i = \pm a_i$ corresponding to the dominant edge behaviour. To fully understand this point, one has to think the current on each strip as the superposition of the currents on each face of the side itself: the dominant edge behavior has to be chosen among such two currents. In addition, the current must be continuous at nodes.

The edge behavior given by (2) results in the following asymptotic behaviour of the Fourier transform of current components [11]:

$$\tilde{J}_i = \frac{1}{2\pi}\int_{-\infty}^{\infty} J_i(x_i)e^{jux_i}\,dx_i \overset{|u|\to\infty}{\sim} j_i^{\infty}(u) + \frac{\eta_j^- e^{-jua_i}}{u^{\pi/(2\pi-\psi_i^-)+1}} + \frac{\eta_j^+ e^{jua_i}}{u^{\pi/(2\pi-\psi_i^+)+1}} \qquad (3)$$

where $\eta_i^{\pm}$ are quantities depending; on the incidence and on the cross-section of the scatterers, and

$$\tilde{J}_i^{\infty}(u) = \frac{-C_i^- e^{-jua_i} + C_i^+ e^{jua_i}}{2\pi\, ju} \qquad (4)$$

In (4) $C_i^{\pm}$ are the values of the current at edges of ith side (right and left edge, respectively). Of course, when $C_i^- = C_i^+ = 0$ that is the case of a single strip, the asymptotic behavior is dominated by the remaining terms in (3). A straightforward consequence of the asymptotic behavior (4) is that the decaying of the integrand in (1) is such that it is always possible to invert one derivative and the integration, obtaining

$$E_j(x,y) = \frac{1}{2j\omega\varepsilon}\frac{\partial}{\partial y_j}\sum_{i=1}^{L}\operatorname{sgn}(y_i)\int_{-\infty}^{+\infty}\hat{J}_i(u)e^{-j|y|\sqrt{x^2-u^2}}e^{-jux_i}\,du \qquad (5)$$

In (5) the direct inversion of the residual derivative and integration is not possible when y_i is zero, and managing with it in order to simplify the equation requires some precaution. A possible way to proceed consists in subtracting and summing the asymptotic behavior of the integrand obtaining

$$E_j(x,y) = \frac{1}{2j\omega\varepsilon}\frac{\partial}{\partial y_j}\left[\sum_{i=1}^{L}\operatorname{sgn}(y_i)\int_{-\infty}^{+\infty}\left[\hat{J}_i(u)e^{-j|y|\sqrt{x^2-u^2}} - \hat{J}_i^{\infty}(u)e^{-|y,u}\right]e^{-jux_i}\,du + \frac{\partial}{\partial y_j}\sum_{i=1}^{L}\operatorname{sgn}(y_i)\int_{-\infty}^{+\infty}\hat{J}_i(u)e^{-|y,u}e^{-jux},du\right] \qquad (6)$$

In this way the first integral can be inverted with the derivative, which it can be demonstrated that the last term in (6) is zero. Finally the electric field is

$$E_j(x,y) = -\frac{1}{2\omega\varepsilon}\sum_{-\infty}^{+\infty}\left[\hat{J}_i(u)G_{ij}(u)e^{-j|y_i|\sqrt{k^2-u^2}}\ \hat{J}_i^{\infty}(u)\tilde{G}_{ij}^{\infty}(u)e^{-|y,u|}\right]e^{-jux_i}\,dx \qquad (7)$$

where

$G_{ij}(u) = \sqrt{k^2-u^2}\cos(\varphi_i-\varphi_j) + u\,\operatorname{sgn}(y_i)\sin(\varphi_i-\varphi_j)$ (8)

$G_{ij}^{\infty}(u) = -j|u|\cos(\varphi_i-\varphi_j) + u\,\operatorname{sgn}(y_i)\sin(\varphi_i-\varphi_j)$ (9)

Expression (7) leads to the desired EFIE, with the constraints that the derivative of the current has to exhibit the edge behavior given by (2) and must be continuous at nodes.

Numerical results

Many different geometries have been tested in order to prove the quick convergence and the accuracy of the method. We introduced the following normalized truncation error:

$$e(N) = \frac{\| J_{N+1} - J_N \|}{\| J_N \|} \qquad (10)$$

where the norm is the usual Euclidean norm and J_N and J_{N+1} are the vectors of all the expansion coefficient of the currents on all the sides evaluated with N and N+1 terms respectively on each side. When interested in other quantities than currents, such as the far field, the same criterion has been used by substituting the current vectors with the quantities to be plotted.

As an example, the normalized truncation error is plotted in Fig.-2 for the "tree-shaped" structure sketched in the inset. As can be seen only 4 terms are needed to achieve an error less than 10^{-2} the error decaying being almost exponential.

In the follbwing the same truncation error has been used, unless otherwise specified. All the simulations have been performed. It is worth noting that the commercial software sued to make the comparisons in the following, exhibit poor performance when a small amount of RAM is available. The proposed method performance, on the contrary, essentially depends only on processors speed, due to the small dimensions of the matrices to be dealt of.

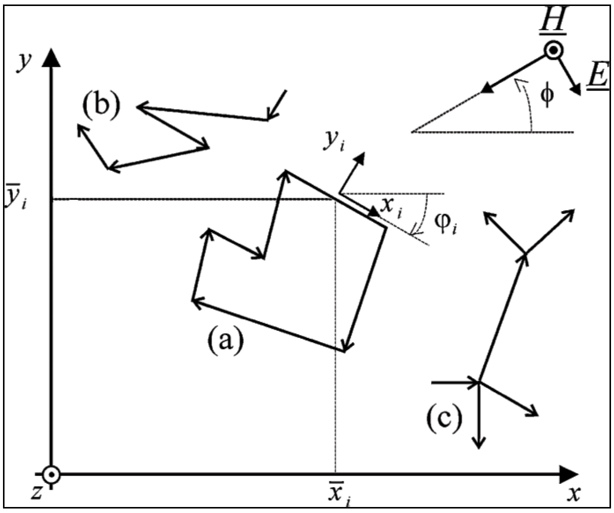

Fig 1: Geometry of the problem: (a) closed cylinder, (b) open cylinder, (c) "tree-shaped" cylinder

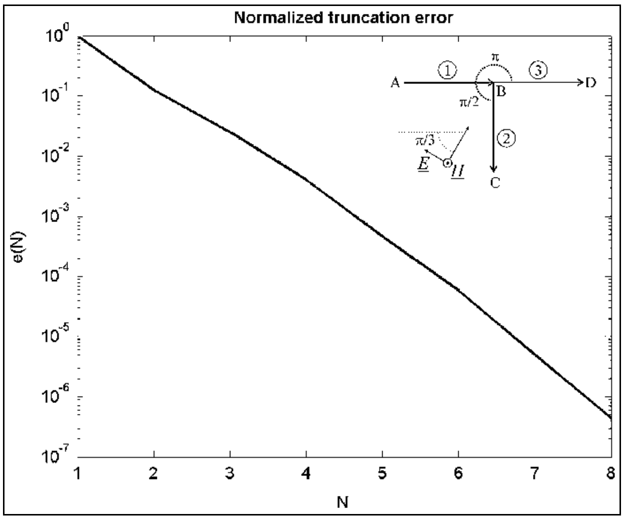

Fig 2: Normalized truncation error for the geometry sketched in the inset

Conclusion

In this communication the scattering by perfectly conducting arbitrarily connected strips for TE incidence has been formulated in the spectral domain in terms of EFIE, in order to include also open scatterers. An effective procedure has been developed, based on the Galerkin method with expansion functions ensuring the correct edge singularity of the electromagnetic field and continuity of the current at edges. In such a way, quick convergence and effective error control can be achieved, even when high accuracy is required. The method is not numerically cumbersome since all the elements of the scattering matrix can be reduced to single integrals, which can be very efficiently computed by means of an analytical accelerating procedure. The method is therefore very attractive, and will be extended in future work to dielectric scatterers.

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