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Study of resonant frequency of equilateral triangular microstrip patch antenna on moment method

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Abstract
In this paper presented about the calculated theoretically the resonant frequencies of the equilateral triangular patch antenna using moment method. Theoretically with the help of curve the effect of substrate thickness on the resonant frequency of an equilateral triangular microstrip patch antenna.

Keywords: Equilateral Triangular and Patch Antenna

Introductions

Circular or rectangular microstrip antennas are widely used to produce linearly polarised waves. Circular or square antennas can be used to produce circular polarization by feeding at orthogonal points with equal amplitude and quadrature phase or by cutting a titled slot in the patch [1]. A slightly rectangular patch or a slightly elliptical shaped patch with a single feed point can also provide circular polarization [2, 3, 4]. This is accomplished by placing the feed point on a diagonal in order to excite orthogonal currents which resonate at slightly different frequencies. By adjustment or the patch length to width ratio, the orthogonal currents can be excited with equal amplitudes and quadrature phase at an intermediate frequency. An analytical model can aid in the determination of the patch aspect ratio for circulars polarization. A resonant circuit model for the input impedance of an elliptical microstrip antenna has been developed [4] by extending the analysis of a circular disc printed circuit antenna [5].

Recently there have been a number of discussions on the resonant frequencies of the equilateral triangular microstrip patch antenna [6, 7]. The discussions were concerned with the various correction factors to be applied to the formula obtained from the cavity model with perfect magnetic walls and how the predicted values using these correction factors compared with the experimental measurements of Dahele and Lee (DL) [8]. DL had concluded that, for their experimental parameters, if the side-length of the triangular patch is replaced by its effective value while leaving the substrate relative permittivity ($\in_r$) unchanged, good agreement between theory and experiment was obtained. This conclusion was also arrived at by Garg and Long (GL) [9, 10] who proposed an alternative expression for the effective side-length. However, in a recent communication [11], Gang argued that effective values should be used both for the side-length and for the relative permittivity. He proposed an integration average procedure for finding the effective relative permittivity. The difference between the values so obtained and $\in_r$ is small when $\in_r$ is close to unity, but increases as $\in_r$ increases. Gang [11] stated that the good agreement obtained by DL [8] between theory and experiment was "by chance" Since under the conditions of the experiment, the difference between $\in_r$ and the effective value calculated by his procedure is small. He implied that for larger values of $\in_r$ e.g. 10, substantial discrepancies between theory and experiment can be expected if effective values of $\in_r$ are not used. Unfortunately, there were no experimental data to test his hypothesis.

In this paper we shall have a critical study of the resonant frequency of the equilateral triangular microstrip antenna on moment method, the theoretical resonant frequency calculated by the moment method will be compared with the experimentally observed values available in literature.

Fundamental idea of moment method

In his theory of gravitation, Newton introduced another concept which deeply influenced physics: the "action at a distance".

Newton was quite aware of the fact that his law gravitation does not explain how the "action" is transported from one body to another, but his integral formulation was very powerful. Once more, it was Faraday who was convinced that the "action at a distance" had to be removed by a "local action". To do this, he invented the concept of the field, but he was not able to give an appropriate mathematical formula i.e., partial differential equations. This was done first in electromagnetics by Maxwell and, finally, in gravitation theory by Einstein. Although the idea of the electromagnetic field is accepted today, the ideas of charges, currents, and forces are more evident for most people. Thus, the electromagnetic field is usually considered to be a fiction rather than a physical reality.

Numerical mathematicians realized very early that the numerical solution of integrals is much easier than be the numerical solution of equations involving differentials. This lead to a revival of integral formulations. Infinite element codes, variational integrals were applied which are based on the energy concept, rather than on the field. These formulations were deeply influenced by the "Maxwellian" pointing. Another kind of integral formulation can be derived from Maxwell's equations by the theory of Green's functions. Such integral equations are preferred in most moment method codes. From the Philosophical point of view, this is a step back to pre-Maxwellian theories, like those of Coulomb, Ampere, and others. Nonetheless, powerful codes can be obtained. Essentially, these integral equations are "action at a distance" formulations. This can be seen from the fact that distances between the locations of the sources and the locations where the field is measured are involved.

It is important to notice that numerical codes based on differential equations can be very efficient. For example, Finite Difference Time Domain codes seem to be best suited to solve electrodynamic problems in the time domain.

The moment method can be defined in a very general sense which makes it difficult to say anything about its analytic background, but most moment method codes are based on integral equations, like the E-field integral equation and the E-field integral equation.

$$E(r)=\frac{-1}{4\pi i\omega\varepsilon'}\int\left[J(r)\left(K^2+\frac{ik}{R}-\frac{1}{R^2}\right)\left[J(r')\frac{R}{R}\right]\left(\frac{R}{R}K^2+\frac{3ik}{R}-\frac{3}{R^2}\right)\right]\frac{e^{ikr}}{R}dv,$$

$$H(r)=\frac{1}{4\pi}\int\left[J(r')\times\frac{R}{R}\right]\left(ik-\frac{1}{R}\right)\frac{e^{ikr}}{R}dv',$$

where

R=r–r', R= |R|

Obviously, this is an "action at a distance formulation: R is the distance between the sources of the field, i.e. the current densities, and the point r where the field is measured. It is important to recognize that not all integral equations represent an "action at a distance." For example, no "distances" like the term R are involved in the integral formulations of Maxwells equations and in variational integrals. The integrals above can be considered to be a generalization of the Coulomb integrals for time - harmonic fields. This leads to a second important fact: Most moment method codes work in the frequency domain, which is reasonable for many applications.

To derive integral equations like the E - field integral equation and the H - field integral equation from Maxwells partial differential equations, several steps are important:

- Introduction of potentials
- Time - harmonic fields or application of fourier analysis.
- Decoupling of first -order partial differential equations.
- Application of Green's functions and theorems.
- It is not necessary to perform the different steps in the sequence given above. For example, one can decouple the first -order partial differential equations before the assumption of time -harmonic fields. Essentially Helmholtz equations of the form

$(\Delta + K^2)\, Z(r) = Y(r)$

are derived, where the complex amplitudes; Z, Y of the field and of the source are either vectors or scalars. The wave number, K, contains the frequency and the material properties. For lossy media, K is complex.

To integrate the Helmholtz equation, the source term, Y, is replaced first by a Dirac function (i.e., by a point source at a point r'):

$(\Delta + K^2)\, G(r, r') = \delta(r - r')$

where G is the Green's function. The problem with this formulation is that δ is not a function in the classical sense, but the procedure is nonetheless successful. Incidentally, δ represents Newton's "mass-point" in mechanics, and the "point charge" in electrostatics. Here, δ is the amplitude of an oscillating point source.

The introduction of a source concentrated in a point has the following advantage: one can expect that a solution with spherical symmetry with respect to the point r_1 exists, because the source δ has this symmetry. Assuming spherical symmetry leads to a one dimensional differential equation for the radial dependence

$$\frac{1}{R}\left[\frac{\partial^2}{\partial R^2}+K^2\right](RG)=\delta(R)$$

and to the physically - meaningful solution.

In order to get a set of scalar equations from any operator equation of the form

Lf =g

where L is a linear operator, f is the unknown function and g is a known function, a symmetric product of functions

$< f_1 f_2 > = < F_2 f_1 >$

is defined by an integral of the form

$< f_1 f_2 = \int f_1, f_2\, dV >$

In the case of real functions, this is a scalar or inner product, which defines the norm

$\|f\| = <f, f>$

of a function f. Thus, multiplying both sides of an operator a equation with a testing function, t_i, leads to the scalar equation

$< Lf, t_i > = < g, >$

if Lf and g are scalars. Introducing a series expansion

$$f = f_0+\eta=\sum_{k=1}^{k}A_k f_k+\eta$$

where the A_K are unknown parameters, the f_K are the basis functions for the approximation f^0 of f, and η is an error function, leads to the linear system of equations.

$$\sum_{k=1}^{k} A_k < Lf_k, t_i > = < g, t_i - \eta_i$$

where η_i is the product of the error η with the testing function, t_i. This system can be solved with $\eta_i = 0$ if the number of equations, I (i.e, testing functions), is equal to the number of unknown parameters, K. Of course, this does not mean that the error of such a computation is zero.

For complex functions, the symmetric product does not define a norm. To get a scalar product which defines a norm, the conjugate complex of one of the functions must be introduced:

$< f_1 f_2 > = \int f_{2*,} f_2^* dV$.

In the moment method, this is considered not to be very important, because one can write

$< f_1 f_2^* > = < f_2 \bullet f_1 >$

Thus, one can get identical results with both definitions. As Sarkar [12] pointed out, the symmetric product arises from reciprocity, where as the Hilbert scalar product describes power. The norm of a function is important for the definition of a scalar error:

$\varepsilon^p = ||F(f, f^0)||^p$,

Where P is the error norm, F an operator; which can be defined in such a way that the minimization of ε^p leads to a linear system of equations. For example, the minimization of the square error (P = 2) with

$F(f, f^0) = L(f, -f^0) = g - Lf^0$,

i.e., the minimization of the square error of the inhomogeneity, g, leads to

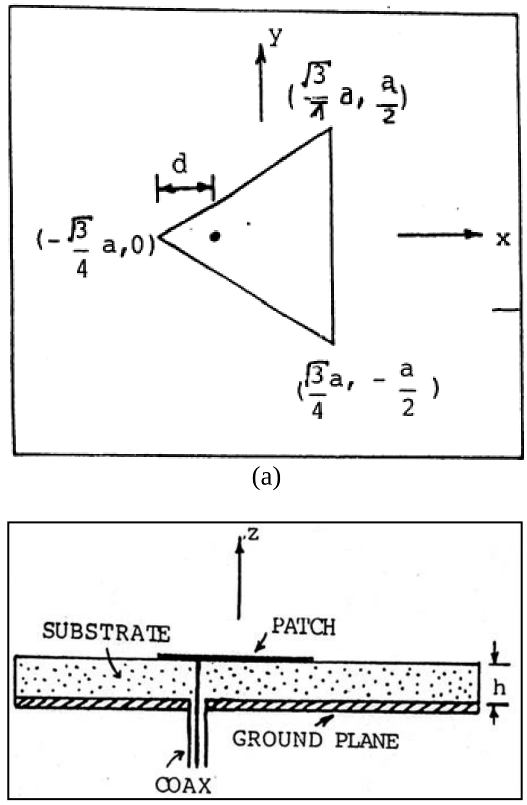
$$\sum_{k=1}^{k} A_k (Lf_k, Lf_i) = (g, Lf_i),$$

Which is equivalent to the use of the testing. Function $t_i = Lf^*$ in the procedure usually applied in the moment method. Besides this, one can derive other equivalences of moment method solutions, with certain testing functions and the minimization of appropriate en-or definitions. Such equivalences are important for finding useful testing functions and for recognizing the error which is actually minimized [13, 14]. From this point of view, the symmetric product looks less natural than the scalar product.

Moment method analysis of resonant frequency of equilateral triangular microstrip patch antenna

The geometry of the probe-fed equilateral microstrip patch antenna is shown in figure l. One of our objectives is to obtain the resonant frequencies using the spectral domain full wave analysis and the moment method. For this purpose, we consider the source-free case and employ the procedure similar to the rectangular patch [15, 16]. By enforcing the boundary condition that the tangential electric field vanishes on the patch and applying Galerkin's method, we arrive at the matrix equation:

$[A_{ij}] [C_n] = 0$ (1)



(a)

(b)

Fig 1: Geometry of the equilateral triangular patch antenna. (a) Top view, (b) Side view.

Where

$$A_{ij} = \frac{1}{(2\pi)^2}\int\int_{-\infty}^{\infty}[Z_{xx} f_{ix} f_{jx}(-K_x,-K_y) + Z_{xy}(f_{iy} f_{jx}(-K_x - K_y)$$
$$+ f_{ix} f_{jy}(-K_x,-K_y)) + Z_{yy} f_{iy} f_{jy}(-K_{x,} - K_y) dk_x dk_y \quad (2)$$

$$J_s = \sum C_n (f_{nx} x + f_{ny} y) \quad (3)$$

The tilde ~ denotes Fourier transform, and J_S in (3) is the surface current density on the patch. The matrix elements Z_{xx}, Z_{xy} and Z_{yy} are given in [16, 17].

The complex resonant frequencies ($f_r + jf_i$) are found by solving the equation

$\det(A_{ij}) = 0$ (4)

In this paper, the entire domain basis functions from cavity modes are used as expansion functions in (3). They are given by

$$f_x(m,n) = \sqrt{3}\left[l \sin\left(\frac{2\pi / x}{\sqrt{3}a}\right)\cos\left(\frac{2\pi(m-n)y}{3a}\right) + m\sin\left(\frac{2\pi mx}{\sqrt{3}a}\right)\cos\left(\frac{2\pi(n-l)y}{3a}\right) + n\sin\left(\frac{2\pi nx}{\sqrt{3}a}\right)\cos\left(\frac{2\pi(l-m)y}{3a}\right)\right. \quad (5a)$$

$$f_y(m,n)=(m-n)\cos\left(\frac{2\pi/x}{\sqrt{3}a}\right)\sin\left(\frac{2\pi(m-n)y}{3a}\right)+(n-l)\cos\left(\frac{2\pi mx}{\sqrt{3}a}\right)\sin\left(\frac{2\pi(n-l)y}{3a}\right)+(l-m)\cos\left(\frac{2\pi mx}{\sqrt{3}a}\right)\sin\left(\frac{2\pi(l-m)y}{3a}\right) \quad (5b)$$

Where $l = -m - n$

It is found that using three modes in the expansion is sufficient to give accurate results. Except in the case of the lowest mode, the three modes used include one higher and one lower than the mode whose resonant frequency is being calculated.

Comparison of moment method results with observed results

Table: 1 Compares the Moment method results, we obtained and the previous experimental data of Dahele and Lee [8] for the resonant frequencies of the first five modes of a probe-fed equilateral triangular patch antenna with a = 10 cm. $\in_r = 2.30$; h = 0.158 cm and d = 3 mm.

Table 1: Compares the Moment method

Mode	Resonant frequencies (GHz)	
	Measured	Moment Method
TM_{10}	1.280	1.291
TM_{11}	2.242	2.262
TM_{20}	2.550	2.613
TM_{21}	3.400	3.457
TM_{30}	3.824	3.878

Modification of the analysis with curve fitting method

According to the cavity model with perfect magnetic walls, the relationship between the resonant frequency of the lowest mode, f_{10}, and an higher order mode f_{10}, is

$f_{mn} = f_{10}\,(m^2+mn+n^2)^{½}$ (6)

It is found that this relationship still holds to an accuracy of about 0.5%. It would therefore be very useful to antenna designers if a curve fitting formula for f_{10} be obtained based on the moment method results. The frequencies of the higher modes can then be obtained from (6). A curve fitting formula for the rectangular patch based on moment method results has been given by Chew and Liu [15] Using the approach of these authors, we arrive at the following formula, which yields resonant frequencies which are within 1% of the moment method values

$$f_{10}=\frac{2C}{3a_e\sqrt{\in_r}} \quad (7)$$

Singh, De and Yadav [10] modified a_{eff} as

$$a'=a\left[1+\frac{2h}{\pi\in_r a_{eq}}\left\{\log\left(\frac{\pi a_{eq}}{2h}\right)+1.726\right\}\right]^{½} \quad (8)$$

Here the effective dimension is estimated using the capacitance formula in [18, 19]

$$C=\frac{a^2\pi\in_r}{h}\left[1+\frac{2h}{\pi\in_r a}\left\{\log\left(\frac{a}{2h}\right)+(1.41\in_r+1.77)+\frac{h}{a}(0.268\pi\in_r+1.65)\right\}\right] \quad (9)$$

This formula can be written as

$$a_{ef}=a\left[1+\frac{2h}{\pi\varepsilon_r a}\left\{\log\left(\frac{a}{2h}\right)+(1.41\in_r+1.77)+\frac{h}{a}(0.268\in_r+1.65)\right\}\right]^{½} \quad (10)$$

Where a and aef are the physical and effective length of the triangular path, respectively. The resonant frequency formula of the equilateral triangular microstrip patch antenna was given by [10]

$$f_{m,n,1}=\frac{2C}{3a(\in_r)^{½}}(m^2+mn+n^2)^{½} \quad (11)$$

2C

Where $\in_r$ is the velocity of light (3 × 10^{10}cm/s). The resonant frequency formula in (11) is modified to account for the edges extension defined by

$$f_{m,n,1}=\frac{2C}{3a_{ef}(\in_r)^{½}}(m^2+mn+n^2)^{½} \quad (12)$$

where $\in_r$ is the relative permittivity of dielectric (substrate).

Variation of resonant frequency with substrate thickness

The computed values of resonant frequency for an equilateral triangular patch with a = 10 cm and etched on substrate with $\in_r$ = 2.32 and several thicknesses where calculated using the method of Moment. The results are shown in table 2.

Table 2: Calculated using the method of Moment

Side of the patch a(cm)	Dielectric Constant $\in_r$	Substrate Thickness h(m)	Resonant Frequency GHz
10	2.32	5×10^{-4}	1.299
10	2.32	1×10^{-3}	1.294
10	2.32	2×10^{-3}	1.284
10	2.32	4×10^{-3}	1.267
10	2.32	8×10^{-3}	1.233
10	2.32	1.6×10^{-2}	1.172
10	2.32	3.2×10^{-2}	1.053

From the calculated resonant frequency on different substrate thickness a curve is plotted which is shown in the figure 2.

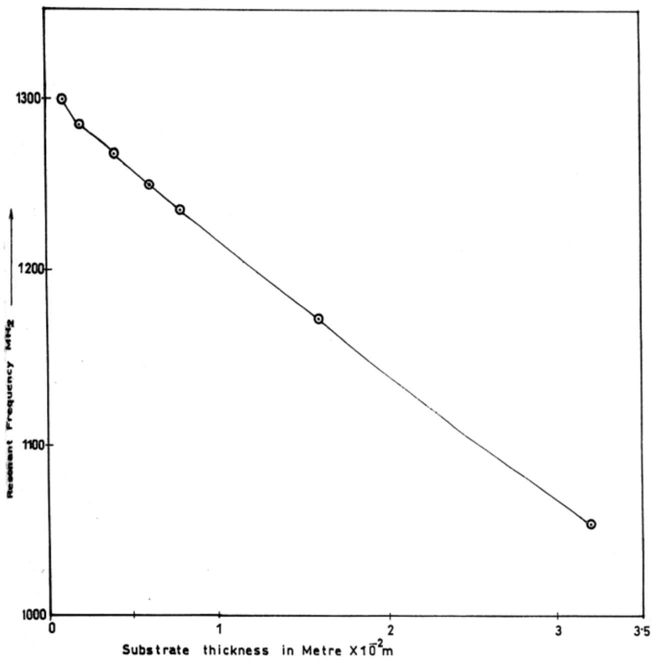


Fig 2: Resonant frequencies of an equilateral triangular microstrip patch antenna

This curve shows the variation of resonant frequencies of an equilateral triangular microstrip patch antenna with substrate thickness. The curve tells us that with the increase of the substrate thickness the resonant frequency decreases and at higher thickness its decrease is asymptotic.

Conclusion

In this paper we have calculated theoretically the resonant frequencies of the equilateral triangular patch antenna using moment method. We have also utilized curve fitting formula already used by Chew and Liu [15]. We have theoretically with the help of curve the effect of substrate thickness on the resonant frequency of an equilateral triangular microstrip patch antenna.

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