Tansient radiation from simple current distributions

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Abstract
In this paper we have considered the transient radiation fr
om two simple, filamentary current
distribution that are frequently used to more than practical antennas as for example the traveling wave
element and the standing dipole. Exact analytical expressions were presented for the electric ad
magnetic fields of these distribution when the excitation was a general function of time. These
expressions apply in both the near and far zone. For an excitation i.e., Gaussian pulse in time, exact
analytical expressions were obtained for the energy leaving the filament per unit time per .unit length,
the total energy leaving the filament per unit length, and the total energy radiated.

Keywords: Current distribution, transient radiation, electromagnetic radiation.

Introduction
During the last few years, a number of articles have appeared in the publications that deal
with the radiation from simple filamentary current distribution and simple wire antennas [1-
6]. The purpose of these articles generally is to add to the physical understanding of the process
of radiation, that is, to add to the understanding of where radiation originates on these
structures, and the mechanism by which energy propagates away from the structures t the far
zone.

In an earlier article the author produced the radiation from two simple filamentary current
distributions: traveling-wave and uniform [3]. The radiated or far-zone electric field was
computed for an excitation that was a Gaussian pulse in time. Two interpretations for the
origin of the radiation were presented, based on the far-field results. In this article, we
continue this investigation; however, the emphasis is on an examination of the near filed and
the related transport of energy away from the current distributions, because these distributors
are frequently used to model practical antennas.

Electromagnetic fields of two filamentary current distributions
The geometry and the associated coordinates for the traveling-wave current distribution,
which we call the traveling-wave element, are shown in Fig. -la. The element, of length h, is
aligned with the z axis. There is source of current I(t) at the bottom of the element, and there
is a perfect termination at the top of the element. A traveling wave of current leaves the
source and propagates along the element at the speed of light, c, unit it reaches the
termination, where it is totally absorbed. The distribution for the axial current is
I(z,t) = I(t-z/c)[U(z) – U(z-h)] ...(1)
and, from the equation of continuity for electric charge, the change per unit length on the
element is
Q(z,t) = Qs(t-z/c)[U(z)–U(z-h)]+Q0(δ(z)+Qh(δ(z-h) δ(z-h)) ...(2)
where
Qs(t) = I(t)/c,
q0(t) = 0 I (t’-z/c) dt’
qh(t) = - 0 I (t’-h/c ) dt’ ...(3)
Here, U is the Heaviside unit-step function, and δ is the Dirac delta function. The three terms
in Equation (2) represent a traveling wave of positive charge, Qs, propagating along the
element at the speed of light; a negative charge, q0,
that is left behind at the lower end as the pulse of positive charge leaves the source (the element is electrically neutral): and a positive charge, \( q_h \), that accumulates at the upper end as the pulse enters the termination.

The geometry and the associated coordinates for the standing-wave current distribution, which we call the standing-wave dipole, are shown in Fig. 1b. This terminology is used because the distribution becomes the familiar standing wave when the excitation is time harmonic. It is a dipole with the arms (each of length \( h \)) aligned with the z axis. There is a source of current, \( L(t) \) at the center of the dipole (\( z=0 \)). The source produces a traveling wave of current (a pulse of positive charge) that propagates at the speed of light up the top arm of the dipole.

A similar traveling wave of current (a pulse of negative charge) propagates down the bottom arm of the dipole. These waves are totally reflected when they reach the open ends of the dipole at time \( t = \tau_0 = h/c \). This produces traveling waves of current that propagate on the arms from the open ends toward the source. These waves are totally absorbed when they reach the source at time \( t = 2\tau_0 = 2h/c \). The distribution for the axial current is

\[
I(z,t)=\left[ I_s(t-z/c) - I_s(t+z/2c - 2h/c) \right] \left[ U(z) - U(z-h) \right] + \left[ I_s(t+z/c) - I_s(t-z/c-2h/c) \right] \left[ U(z+h) - U(z) \right], \tag{4}
\]

and the charge per unit length on the dipole is

\[
Q(z,t)=\left[ Q_s(t-z/c) - Q_s(t+z/2c - 2h/c) \right] \left[ U(z) - U(z-h) \right] + \left[ Q_s(t+z/c) - Q_s(t-z/c-2h/c) \right] \left[ U(z+h) - U(z) \right], \tag{5}
\]

The standing-wave dipole can be viewed as a combination of four basic traveling-wave elements. Two elements are arranged to produce outward-traveling waves on the arms, starting at time \( t = 0 \), and two elements are arranged to produce inward-traveling waves, starting at time \( t = \tau_0 = h/c \); this representation is easily understood by comparing the current distributions given in Equations (1) and (4): Equation (4) is the sum of four terms, each with the same form as Equation (1). Notice from Equation (5) that there is no accumulation of charge at the center or ends of the standing-wave dipole as there is for the traveling-wave element of Equation (2). For the standing-wave dipole, equal amounts of positive and negative charge simultaneously leave or enter the source, and the traveling waves of charge are totally reflected at the open ends.

The complete electromagnetic field (both the near field and far field) of these current distributions can be obtained in closed form [7]. The field for the traveling-wave element is

\[
\vec{E}(r,t) = \frac{1}{4\pi} \left[ \frac{q_0(t-r/c)}{r^2} \hat{r} + \frac{q_h(t-r_h/c)}{r_h^2} \hat{r}_h \right. \\
\left. + \frac{\cot(\theta/2)I_s(t-r/c) \hat{\phi}}{cr} - \frac{\cot(\theta/2)I_s(t-h/c-r_h/c) \hat{\phi}}{cr_h} \right], \tag{6}
\]
This expression can be evaluated in closed form for
\[ \sin \mu \frac{H_0}{2r} \left[ I_x(t-r/c) - I_x(t-h/c-r_h/c) \right] \phi, \]
and the field for the standing-wave dipole is
\[ E(r,t) = \frac{\mu_0 c}{2\pi \sin \theta} \left\{ I_x(t-r/c) + I_x(t-2h/c-r/c) \phi \right\} \]
\[ - I_x(t-h/c-r_h/c) \phi_h - I_x(t-h/c-r_h/c) \phi_h, \]
\[ B(r,t) = \frac{\mu_0 c}{2\pi \sin \theta} \left\{ I_x(t-r/c) + I_x(t-2h/c-r/c) \phi \right\} \]
\[ - I_x(t-h/c-r_h/c) - I_x(t-h/c-r_h/c) \phi_h, \]
where \[ \phi_h = \phi_h = \phi \]
In the limit as \( r \rightarrow \infty \), Equations (6)–(9) simplify to become the radiator far-zone field. For the traveling-wave element the electric field is
\[ \dot{E}^r(r,t) = \frac{\mu_0 c \sin \theta}{4\pi (1-\cos \theta)} \left\{ I_x(t-r/c) - I_x(t-r/c-(h/c)(1-\cos \theta)) \right\} \phi \]
and for the standing-wave dipole, the electric field is
\[ \dot{E}^s(r,t) = \frac{\mu_0 c \sin \theta}{4\pi (1-\cos \theta)} \left\{ I_x(t-r/c) - I_x(t-r/c-2h/c) \right\} \phi \]
\[ - I_x(t-r/c-(h/c)(1-\cos \theta)) - I_x(t-r/c-(h/c)(1+\cos \theta)) \phi \]
for both distributions, the radiated magnetic field is simple
\[ \dot{B}^r(r,t) = \frac{1}{c} \hat{r} \times \dot{E}^r(r,t) \]
notice that the superscript r is used to indicate the radiated or far-zone field.
In the calculations that follow, the current of the source is assumed to be a Gaussian pulse of the form
\[ I_x(t) = I_0 e^{-(t/\tau)^2} \]
where \( \tau \) is the characteristic time. For all numerical results, we will use \( \tau/\tau_0 = 0.076 \); then, the width of the pulse in space is approximately one fourth of the length of an element (four pulses fit along the length h).
The expressions for the electric and magnetic fields of the current distributions, Equations (6)–(9), apply at any point not directly on the filament. Therefore, we can use these expressions to calculate the Poynting vector in the space surrounding the filament:
\[ S(r,t) = \frac{1}{\mu_0} \dot{E}^r(r,t) \times \dot{B}(r,t) \]
Total energy radiated by the filamentary current distributions
The total energy radiated by a filament is determined by integrating the normal component of the Poynting vector for the radiated field, \( \dot{S}^r \), over the surface of a large sphere surrounding the filament and over all time:
\[ U_{md} = 2\pi \int_{r=0}^{r} \int_{0}^{2\pi} r' \sin \theta \hat{r} \cdot \dot{S}^r r^2 \sin \theta \hat{r} dl \hat{r} dt = \frac{2\pi}{\hat{r}_0} \int_{r=0}^{r} \int_{0}^{2\pi} |\dot{E}^r|^2 r^2 \sin \theta \hat{r} dl \hat{r} dt \]
where \( \hat{r}_0 = \sqrt{\mu_0 / \varepsilon_0} \) is the wave impedance of free space. Suprisingly, this expression can be evaluated in closed form for both current distributions when the source current is the Gaussian pulse of Equation (13) (see Appendix A). For the traveling-wave element, we obtain

\[ \text{Total energy radiated by the filamentary current distributions} \]
\[ U_{nd} = \frac{\varepsilon_0 \mu_0^2}{4\sqrt{2\pi}} \left\{ \gamma - 2 + \ln\left[2(\tau_a / \tau)^2\right] \right. \\
+ \sqrt{\frac{\pi}{2}} \left( \frac{\tau}{\tau_a} \right) \text{erf}\left[\sqrt{2}(\tau_a / \tau)\right] + E_i\left[2(\tau_a / \tau)^2\right] \right\} \]

and for the standing-wave dipole
\[ U_{nd} = \frac{\varepsilon_0 \mu_0^2}{\sqrt{2\pi}} \left\{ \left( \gamma + \ln\left[2(\tau_a / \tau)^2\right]\right)[1 + \exp\left(2(\tau_a / \tau)^2\right] \right\} \\
+ E_i\left[2(\tau_a / \tau)^2\right] - \exp\left(2(\tau_a / \tau)^2\right)E_i\left[2(\tau_a / \tau)^2\right] \]

equation for the two structures are very similar. So, we can conclude that even though energy leaves the surfaces of the two structures in completely different ways, once we are slightly away from the structures—at a radius that is typically 10% to 20% of the height of the monopole—the distribution of the energy in space is very similar for the two structures.

**Conclusions**

There are several differences in the descriptions presented above for the energy transport very close to these two structures. Probably the most distinct difference is that energy is continually exchanged between the current filament and the spherical wave-fronts as they travel along the filament, whereas no energy enters or leaves a spherical wave front through the surface of the perfectly conducting monopole. Energy enters the space surrounding the antenna only through the coaxial aperture, and the perfectly conducting monopole simply serves to distribute this energy in space. Despite these differences, at a small distance from the structures (typically 10% to 20% of the length h) the energy transport is conspicuously similar. This is the reason that the simple model (a filamentary current) is useful for making good qualitative predictions for the radiations (far field) of the actual antenna (a monopole).

**References**