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Correlation between direct product and sylows theorem on automorphic image of permutation group

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Abstract
We should know that the product of two sets as a set of ordered pairs. We now search a new group through the product of two groups.
Let G_1, G_2 be any two subgroups
Let $G = G_1 \times G_2 = [(g_1, g_2): g_1 \in G_1$
What better way could $g_2 \in G_2\}$ there be than to define multiplication on G by $(g_1, g_2)\ (g_1' g_2') = (g_1\ g_1', g_2 g_2')$. That G forms a group under this as its composition should not be a difficult task for the reader. We focus on the matter that of G is a group of order 15; it is IDp (interval direct product) of its sylow subgroups.

Keywords: IDP, composition

Introduction
There can't exist any tow homomorphism from $Z_{16} \times Z_2 \rightarrow Z4 \times Z4$. If N be the normal subgroup in G if G= H×K I where H, K are subgroups of G. then it is either N is abelian or N intersects H or K non- trivially. If in and *n* be relatively prime integer if binary composition of G is multiplication. Then the subgroup G [n] will be $[x \in G | x^n = e\}$ and we can denote it by G_n. Again the subgroups nG will be $\{x^n | x \in G\}$ and it we can denote it as G^n. So $\frac{G}{Gn} \cong G^n$ it can be good exercise for us to the above things under the multiplication composition:

Theorem 1: Let G be a finite abelian group of order p^n, p a prime, Suppose $G = A_1 \times A_2 ....\times A_k$, where each Ai is a cyclic group of order p^{n_i} with $n_1 \geq n_2 \geq .... \geq n_k \geq 0$. Then integers $n_1 ....n_k$ are uniquely determined called invariant

Problem 1: Suppose G is an abelian group of order 120 and suppose G has exactly three elements of order 2. Find the isomorphism class of
G.

Soln: $O(G) = 120 = 2^3 \times 3 \times 5$. So the number of isomorphism class of G is p(3) p(1) p(1) = 3×1×1 =3 and these are $Z_2 3 \times Z_3 1 \times Z_5 1 = Z_8 \times Z_3 \times Z_5 = Z\ 120$.
$Z_4 \times Z_2 \times Z_3 \times Z_5$, $Z_2 \times Z_2 \times Z_2 \times Z_3 \times Z_5$
$Z_8 \times Z_{3\times} Z_5$ has only one element (4, 0, 0) of order 2 so it can't be G.
Again $Z_2 \times Z_2 \times Z_2 \times Z_3 \times Z_5$ has (1, 1, 1, 0, 0), (1, 0, 1, 0, 0), (0, 1, 1, 0, 0) and (1, 1, 0, 0, 0) as elements of order 2. So it cant'be G where as $Z_4 \times Z_2 \times Z_3 \times Z_5$ has exactly three elements (2, 1, 0, 0,), (0, 1, 0, 0), (2, 0, 0, 0) which have order 2 hence G is $Z_4 \times Z_2 \times Z_3 \times Z_5$

Lemma: Let G be a finite group under addition, Let n be a positive integer St. nG = {nx: x∈G} and G [n] = {x∈G : nx = 0} then nG and G[N] are subgroups of G and $\frac{G}{G[n]} \cong nG$.

Aim: Since Z_n has an elements of order n and G has no element of order n. Therefore G is not Z_n.
Again $Z_2 \times Z_2 \times Z_2$ has no element of order 4 and so we are left with the only choice that G is $Z_2 \times Z_2$ to write G as IDP of cyclic groups, we pick up an element of maximum order 4. As we know if G be an abelian group of Prime power order p^n and let $a \in G$ has maximal order

amongst all elements in G. Then G is IDP of A and K where A is the cyclic subgroup generated by a and K≤G. Hence G can be expressed as G = A×K.

Conclusion

A finite abelian group is direct product of cyclic group of prime power order.

S_1 = is IDP of A_L and K_i, S_L= Ai ki, $A_1 \cap ki$ =< e}

$\therefore$ O (S_i) = $\frac{O(A_i)O(K_i)}{O(A_1 \cap K_i)}$ = O(A_i)O(K_i). But O (S_i) = $P_i^{\alpha i}$= Prime order and Thus O(A_i) and O (K_i) being its divisors are also prime powers summing up we notice that any finte abelian group is product of S_1,S_2,........,S_n. Where each S_i is a group of prime power. Order and each S_i is Then a product of cyclic group of Prime power order.

If n = $P_1^{\alpha 1}$ $P_2^{\alpha 2}$... $P_r^{\alpha r}$. Where P_i^S are distinct primes. Then the number of non- isomorphic abelian group of order *n* is P (α_1) P (α_2).....P(α_r) where P (α_i) denotes the number of partition of α_i. If G be the finite abelian group of order mp^n. Where p×m. Then G is IDP of H and K. Where H = {X∈G:|xP^n=e} and K= {x∈G|x^m=e}. two abelian groups of order p^n are isomorphic if and only if they have the same invariants. The no of non isomorphic abelian group of order p^n. P a prime equals the number of partitions on *n*.

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