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# Probabilistic analysis of a human system with maximum repair time of renal Failure by RPGT

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**Abstract**
The objective of this paper is to analyze probabilistically of key performance measures of a human system comprising two identical kidneys, focusing on the repair process before a pre-determined time 't'. The system continues to work effectively as long as at least one kidney is functioning well. Immediate medical interventions, such as dialysis or other procedures, are assumed to be available. If a kidney is not repaired within the maximum allowable time, it is replaced. The study uses the concept of a base state to evaluate the sensitivity of the affected individual's organ capacity factor, along with a fuzziness measure for the renal system, both considered to be statistically independent. The time-to-failure, time to repair, and time to replacement follow a negative exponential distribution, while the actual durations for repair and replacement are treated as arbitrary distributions. To derive steady-state expressions for system performance metrics, the analysis employs a semi-Markov process along with a regenerative point graphical technique. A specific case is presented to illustrate the behavior of metrics such as the mean time to system failure (MTSF), availability, and the busy periods associated with different functional states.

**Keywords:** RPGT, base-state, capacity factor, fuzziness measure

## 1. Introductions

In today's fast-paced lifestyle, people of all ages are increasingly affected by lifestyle-related diseases, with renal failure emerging as one of the major health concerns. The kidneys play a crucial role in maintaining health of body by removing waste products and excess fluids. When kidney function deteriorates, the body's efficiency is significantly compromised, posing a serious health risk if not treated in a timely manner. To support longevity and well-being, treatment options such as dialysis or kidney transplantation are often employed. Additionally, proper care involves strict adherence to prescribed medications and personalized dietary guidelines. Regular follow-up appointments are also essential to ensure comprehensive treatment and monitoring.

The kidneys are vital organs responsible for filtering waste products from the blood, including nitrogenous waste and other substances like medications. They also regulate electrolyte levels, contribute to the production of erythropoietin hormone, and help metabolize low molecular weight proteins such as insulin. Almirall J (2016) [9] has described failure of the kidneys to perform these functions results in the accumulation of waste products in the bloodstream a condition known as renal failure. According to Lindner A, (1996) [8] has categorized Renal failure into two categories.

- **Acute Renal Failure (ARF):** ARF occurs suddenly due to factors like reduced blood supply to the kidneys or toxic overload, which is usually reversible if treated promptly. It is particularly dangerous as it increases probability of diseases like anemia, high blood pressure, high infection and more frequently affects older individuals, with higher prevalence in men than in women.
- **Chronic Renal Failure (CRF):** This form represents the gradual loss of kidney function over time. It is diagnosed when serum creatinine levels remain elevated for at least three months or when the glomerular filtration rate (GFR) drops below 60 ml, advanced stages require ongoing dialysis or a kidney transplant and are classified as End-Stage Renal Disease (ESRD).

From a modeling perspective, immediate intervention is assumed to be available for repair when necessary. If the repair cannot be completed within a specified maximum repair time, the failed kidney is replaced. The time to failure and the time to initiate replacement follow a negative exponential distribution, while repair and replacement durations follow arbitrary distributions with distinct probability density functions. All random variables are considered mutually independent.

Using semi-Markov process and regenerative point graphical techniques, expressions for several reliability measures have been derived in the steady state. These include transition probabilities, mean sojourn times, mean time to system failure (MTSF), availability, busy periods related to repair and replacement, expected number of unit replacements, expected number of maintenance visits, and overall working life. The behavior of these reliability characteristics has been analyzed numerically for various parameter values and associated costs.

## 2. Notations

| | | |
|---|---|---|
| $\exists$ | : | Constant non-working of the kidney |
| $\aleph_0$ | : | The rate by which kidney undergoes for replacement (maximum allowed time) |
| $\rho(t)$ | : | pdf of the replacement time of the kidney |
| $\sigma(t)$ | : | pdf of the repair time of the kidney |
| $\vartheta_i$ | : | The mean sojourn time in state $S_i$ which is given by $\vartheta_i = E(T) = \int_0^\infty P(T > t)\, dt = \sum_j m_{ij}$, Where $T$ denotes the time to kidney failure. |
| $f_i$ | : | Fuzziness measure at ith state. |
| $\eta_i$ | : | Expected time spend while doing a job, given that the system entered state i at time t=0 |
| $J_i$ | : | Probability that the server is busy in state $S_i$ up to time t without making transition to any other regenerative state or returning to the same via one or more regenerative states. |
| $FU_r / FU_R$ | : | kidney is failed and under repair / under repair continuously from previous state |
| $FW_r / FW_R$ | : | kidney is failed and waiting for repair / waiting for repair continuously from previous state |
| $FUR_p / FUR_P$ | : | kidney is failed and under replacement / under replacement continuously from previous state |
| $m_{ij}$ | : | Contribution to mean sojourn time in state $S_i \in E$ and non-regenerative state if occurs before transition to $S_j \in E$. Mathematically, it can be written as $m_{ij} = \int_0^\infty t\, d\left(Q_{ij}(t)\right) = -q_{ij}^{*\prime}(0)$ |

$Stage0 = (O,O), Stage1 = (O,FU_r), Stage2 = \left(O,FU_{R_p}\right), Stage3\left(FW_r,FU_{R_p}\right),$

$Stage4 = (FW_r,FU_R)$ and $Stage5 = (FW_R,FU_{R_p})$. The states $0$, $1$ and $2$ are regenerative while remaining are non-regenerative as shown in Figure 1.

**Model 2.1**

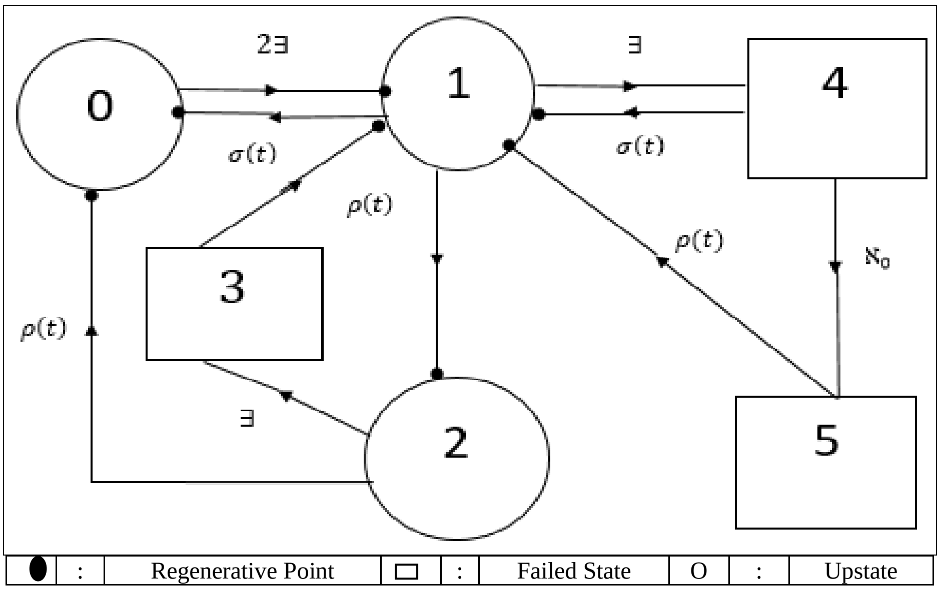


**Fig 1:** State Transition Diagram

**3. Transition probabilities**

Simple probabilistic considerations yield the following expressions for the non-zero elements.

| | |
|---|---|
| $p_{ij} = \int_0^\infty q_{ij}(t) = Q_{ij}(\infty)$; | $\rho(t) = \varphi e^{-\varphi t}$ ; $\sigma(t) = \delta e^{-\delta t}$ |
| $p_{01} = \rho^*(0) = p_{31} = p_{51}$ | $p_{01} = 1 = p_{31} = p_{51}$ |
| $p_{10} = \sigma^*(\exists + \aleph_0)$ | $p_{10} + p_{12} + p_{14} = 1$ |
| $p_{12} = \frac{\aleph_0}{\aleph_0 + \exists}[1 - \sigma^*(\exists + \aleph_0)]$ | |
| $p_{14} = \frac{\exists}{\aleph_0 + \exists}[1 - \sigma^*(\exists + \aleph_0)]$ | |
| $p_{11.4} = \frac{\exists}{\aleph_0 + \exists}[1 - \sigma^*(\exists + \aleph)]\sigma^*(\aleph_0)$ | $p_{10} + p_{12} + p_{11.4} + p_{11.45} = 1$ |
| $p_{11.45} = \frac{\exists}{\aleph_0 + \exists}[1 - \sigma^*(\exists + \aleph_0)][1 - \sigma^*(\aleph_0)]\rho^*(0)$ | |
| $p_{20} = \rho^*(\exists)$ | $p_{20} + p_{23} = 1$ |
| $p_{23} = 1 - \rho^*(\exists)$ | |
| $p_{21.3} = 1 - \rho^*(\exists)$ | $p_{20} + p_{21.3} = 1$ |
| $p_{41} = \sigma^*(\aleph_0)$ | $p_{41} + p_{45} = 1$ |
| $p_{45} = 1 - \sigma^*(\aleph_0)$ | |

**4. Mean sojourn times**

The mean sojourn times $\vartheta_i$ in state $stage\ i$ is given by

| |
|---|
| $\vartheta_i = \int_0^\infty P(T > t)\, dt$ |
| $\vartheta_0 = \mathrm{m}_{01} = \frac{1}{2\exists}$ |
| $\vartheta_1 = \mathrm{m}_{10} + \mathrm{m}_{12} + \mathrm{m}_{14} = \frac{1}{\aleph_0 + \exists}[1 - \sigma^*(\aleph_0 + \exists)]$ |
| $\vartheta_2 = \mathrm{m}_{20} + \mathrm{m}_{23} = \frac{1}{\exists}(1 - \rho^*(\exists))$ |
| $\vartheta_1{}' = \mathrm{m}_{10} + \mathrm{m}_{12} + \mathrm{m}_{11.4} + \mathrm{m}_{11.45} = \frac{[1 - \sigma^*(\aleph_0 + \exists)]}{\aleph_0 + \exists}\left[1 + (1 - \sigma^*(\aleph_0))\left(\frac{\exists}{\aleph_0} - \rho^{*\prime}(0)\right)\right]$ |
| $\vartheta_2' = \mathrm{m}_{20} + \mathrm{m}_{21.3} = (1 - \rho^*(\exists))\left(\frac{1}{\exists} - \rho^{*\prime}(0)\right)$ |

**5. Mean time to system failure (MTSF)**

The regenerative un-failed states to which the system can transit before entering any failed state are i=0,1,2.

$$\mathrm{MTSF} = \frac{\mathrm{N}_1}{\mathrm{D}_1} = \frac{(0-0)\vartheta_0 + (0-1)\vartheta_1 + (0-2)\vartheta_2}{1-(1-0-1)-(1-2-0-1)} = \frac{(0,0)\vartheta_0 + (0,1)\vartheta_1 + (0,1,2,0)\vartheta_2}{1-(1,0,1)-(1,2,0,1)}$$

Where,

$N_1 = \vartheta_0 + \vartheta_1 + p_{12}\vartheta_2$ and $D_1 = 1 - p_{10} - p_{12}p_{20}$.

$$N_1 = \frac{1}{2\exists} + \frac{\aleph_0 + \varphi + \exists}{(\varphi + \exists)(\delta + \exists + \aleph_0)} \quad D_1 = \frac{1}{(\delta + \exists + \aleph_0)}\left[(\exists + \aleph_0) - \frac{\aleph_0 \varphi}{(\varphi + \exists)}\right]$$

**Table 1:** Probabilistic on MTSF Vs. Replacement Rate $(\varphi)$

| $\varphi$ | $(\delta = 3.1, \aleph_0 = 4, \exists = .01)$ | $(\delta = 3.1, \aleph_0 = 4, \exists = .02)$ | $(\delta = 3.1, \aleph_0 = 5, \exists = .01)$ | $(\delta = 3.2, \aleph_0 = 4, \exists = .01)$ |
|---|---|---|---|---|
| 5.1 | 101.9940779 | 51.99859649 | 102.0495598 | 102.022124 |
| 10.2 | 102.5542963 | 52.55859353 | 102.7219954 | 102.5902217 |
| 15.3 | 102.818594 | 52.82293996 | 103.0567233 | 102.8582367 |
| 20.4 | 102.9724519 | 52.97687142 | 103.2570858 | 103.0142585 |
| 25.5 | 103.0731294 | 53.07761518 | 103.3904638 | 103.1163521 |
| 30.6 | 103.1441361 | 53.14867707 | 103.4856375 | 103.1883574 |
| 35.7 | 103.1969038 | 53.20149043 | 103.5569651 | 103.2418673 |
| 40.8 | 103.2376601 | 53.24228469 | 103.612411 | 103.2831968 |
| 45.9 | 103.2700872 | 53.27474359 | 103.6567482 | 103.3160799 |
| 51 | 103.2965015 | 53.30118502 | 103.6930111 | 103.3428658 |

### 6. Availability

The regenerative state at which system is available are i=0, 1, 2, and j=0, 1, 2. Base state=1

$$A_2 = \frac{N_2}{D_2}$$

Where $N_2 = \{(1,0) + (1,2,0)\} f_0 \vartheta_0 + (1-1) f_1 \vartheta_1 + (1,2) f_2 \vartheta_2$

$$D_2 = \{(1,0) + (1,2,0)\}\vartheta_0 + (1-1)\vartheta_1' + (1,2)\vartheta_2'$$

$$N_2 = \vartheta_0(p_{10} + p_{12}p_{20}) + \vartheta_1 + p_{12}\vartheta_2;\ D_2 = \vartheta_0(p_{10} + p_{12}p_{20}) + \vartheta_1' + p_{12}\ \vartheta_2'$$

$$N_2 = \frac{1}{\delta + \exists + \aleph_0}\left[1 + \frac{1}{2\exists}\left(\delta + \frac{\aleph_0 \varphi}{(\varphi + \exists)}\right) + \frac{\aleph_0}{(\varphi + \exists)}\right]$$

$$D_2 = \frac{1}{\delta + \exists + \aleph_0}\left[1 + \frac{\aleph_0}{\delta + \aleph_0}\left(\frac{\exists}{\aleph_0} + \frac{1}{\varphi}\right) + \frac{1}{2\exists}\left(\delta + \frac{\aleph_0 \varphi}{\emptyset + \exists}\right) + \frac{\aleph_0}{\varphi + \exists}\left(\frac{1}{\exists} + \frac{1}{\varphi}\right)\right]$$

**Table 2:** Probabilistic on availability vs. replacement rate $(\varphi)$

| $\varphi$ | $(\delta = 3.1, \aleph_0 = 4, \exists = .01)$ | $(\delta = 3.1, \aleph_0 = 4, \exists = .02)$ | $(\delta = 3.1, \aleph_0 = 5, \exists = .01)$ | $(\delta = 3.2, \aleph_0 = 4, \exists = .01)$ |
|---|---|---|---|---|
| 5.1 | 0.994945904 | 0.990140845 | 0.994571622 | 0.994957934 |
| 10.2 | 0.996282106 | 0.992773246 | 0.99600883 | 0.996297003 |
| 15.3 | 0.996908673 | 0.994001243 | 0.996718787 | 0.996924847 |
| 20.4 | 0.997272301 | 0.994712056 | 0.997142034 | 0.997289197 |
| 25.5 | 0.997509798 | 0.995175578 | 0.997423071 | 0.997527158 |
| 30.6 | 0.997677091 | 0.995501735 | 0.997623263 | 0.997694774 |
| 35.7 | 0.9978013 | 0.995743709 | 0.997773107 | 0.997819221 |
| 40.8 | 0.99789717 | 0.995930367 | 0.997889476 | 0.997915273 |
| 45.9 | 0.997973406 | 0.996078731 | 0.99798246 | 0.997991653 |
| 51 | 0.998035479 | 0.99619949 | 0.998058464 | 0.998053844 |

### 7. Busy period

**The following are the repair / replacement service for kidney:-**

- **Hemo-dialysis:** In this dialysis, a machine is used to move blood through a filer outside the body to remove wastes (type1).
- **Peritoneal dialysis:** It uses the lining of belly to filter blood inside the body to remove wastes (type1).
- **Kidney Transplant:** It is surgery to place a healthy kidney form a person who has just died or from a living person, into patient body (type2).
- **Conservative management:** it treats kidney failure without dialysis or a transplant. Patient work with health care team to manage symptoms and preserve its kidney function and quality of life as long as possible (type1).

Dialysis only replaces part of patient kidney function, while Hemodialysis and peritoneal dialysis allow people with kidney failure to feel better, but neither replaces all of the jobs that healthy kidneys do. Over the years, kidney disease can cause other problems, such as Heart diseases, bone diseases, arthritis, nerve damage, infertility and malnutrition.

**7. A) Due to problem (Type1)**
The regenerative state while doing kidney repair is i=1 and j=0,1,2

$$B_0 = \frac{N_l}{D_2} \; l = 3$$

Where,
$N_3 = P_{10} L_1^*(0)$ and $D_2$ is already specified.

$$N_3 = \frac{1}{\delta + \exists + \aleph_0}[1 + \exists]$$

**7. B). Due to problem (Type2)**
The regenerative state while doing replacement is i=2 and j=0,1,2.

$$B_0 = \frac{N_l}{D_2} \; l = 4$$

$N_4 = P_{12} L_2^*(0)$ and $D_2$ is already specified

$$N_4 = \frac{1}{\exists + \delta + \aleph_0}\left[\frac{\aleph_0(1 + \exists)}{\varphi + \exists}\right]$$

**8. (A) Expected number of visits (Type1)**

The regenerative state where the server visits(afresh) are i=0( repair) and j=0,1,2

$$V_0 = \frac{N_l}{D_2} \; l = 5$$

Where,

$N_5 = P_{10} + P_{12}P_{20}$ and $D_2$ is already specified.

$$N_5 = \frac{1}{\delta + \exists + \aleph_0}\left[\delta + \frac{\aleph_0 \varphi}{\varphi + \exists}\right]$$

**8. (B) Expected number of visits (Type2)**
The regenerative state where the server visits i=2(for replacement) and j=1,2,3

$$V_0 = \frac{N_l}{D_2} \; l = 6$$

Where,

$N_6 = P_{12}$ and $D_2$ is already specified

$$N_6 = \frac{\aleph_0}{\exists + \delta + \aleph_0}$$

**9. Working life:** Working capacity to the kidney system model in steady state is given by:

$$W_0 = K_1 A_0 - K_2 B_0^{t1} - K_3 B_0^{t2} - K_4 V_0^{t1} - K_5 V_0^{t2}$$

| | | |
|---|---|---|
| $K_1$ | = | Health per unit uptime of the kidney system |
| $K_2$ | = | Per unit time when system is busy in type1 problem |
| $K_3$ | = | Per unit time when system is busy in type2 problem |
| $K_4$ | = | Per unit time visits for treatment for type1 |
| $K_5$ | = | Per unit time visits for treatment for type2 |

**Table 3:** Probabilistic on working life Vs. Replacement Rate $(\varphi)$

| $\varphi$ | $(\delta = 3.1, \aleph_0 = 4, \exists = .01)$ | $(\delta = 3.1, \aleph_0 = 4, \exists = .02)$ | $(\delta = 3.1, \aleph_0 = 5, \exists = .01)$ | $(\delta = 3.2, \aleph_0 = 4, \exists = .01)$ |
|---|---|---|---|---|
| 5.1 | 4385.517 | 4385.052 | 4323.643 | 4382.935 |
| 10.2 | 4385.815 | 4387.864 | 4319.484 | 4383.043 |
| 15.3 | 4386.009 | 4389.698 | 4316.723 | 4383.113 |
| 20.4 | 4386.146 | 4390.988 | 4314.756 | 4383.162 |
| 25.5 | 4386.248 | 4391.946 | 4313.283 | 4383.199 |
| 30.6 | 4386.327 | 4392.684 | 4312.14 | 4383.227 |
| 35.7 | 4386.389 | 4393.271 | 4311.226 | 4383.25 |
| 40.8 | 4386.44 | 4393.749 | 4310.479 | 4383.268 |
| 45.9 | 4385.517 | 4385.052 | 4323.643 | 4382.935 |
| 51 | 4385.815 | 4387.864 | 4319.484 | 4383.043 |

**10. Conclusion**
This study incorporates a wide range of potential risk and preventive factors related to kidney function with the findings emphasize the significance of coordinated monitoring of kidney performance under various health conditions. To provide a clearer and more detailed analysis, tables have been included that illustrate reliability metrics such as Mean Time to System Failure (MTSF), system

availability, and operational life. These are plotted against the replacement rate (φ), based on specific parameter values and associated costs, as shown in Figures 2.3 to 2.5.The results indicate that these reliability measures tend to increase with a rise in the replacement rate (φ), the replacement initiation rate ($\aleph_0$), and the repair rate (δ), assuming all other parameters remain constant. Conversely, the measures show a decreasing trend as the failure rate (∃) increases. Among all influencing factors, the repair rate (δ) appears to have the most substantial effect on the performance indicators**.** Furthermore, the analysis shows that the system's efficiency diminishes when the cost of replacing a unit is higher than that of repairing it. Therefore, the study concludes that a kidney system designed to replace a failed unit after a maximum allowable repair time can be made more dependable and efficient either by improving the repair rate (δ) or by shortening the replacement time.

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